

# A Review on Importance of Improving Spectral Efficiency in Massive MIMO Communications

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**Abstract**— Much higher area data throughput is required in future cellular networks, since the global demand for wireless data traffic is continuously growing. This goal can be achieved without the need for more bandwidth or additional base stations if the spectral efficiency is improved. This paper explains why the Massive MIMO (multiple-input multiple-output) communication technology, where multi-antenna base stations spatially multiplex a multitude of user terminals over the entire bandwidth, is well-suited for this purpose.

**Keywords**— Spectral Efficiency, MIMO, CSI, Linear processing.

## I. INTRODUCTION

Massive MIMO is a multi-user MIMO system with  $M$  antennas and  $K$  users per BS. The system is characterized by  $M \gg K$  and operates in time division duplex (TDD) mode using linear uplink and downlink processing. This Massive MIMO technology can be improvements in area throughput by increasing the spectral efficiency (SE) (bit/s/Hz/cell), while using the same bandwidth and density of base stations (BS) as in current networks. These extraordinary gains are achieved by equipping the BS with arrays of a hundred antennas to enable spatial multiplexing (SM) of tens of user terminals (UT).

In contrast, SE has not seen any major improvements in previous network generations. Hence, it might be a factor that can be greatly improved in the future and possibly become the primary way to achieve high area throughput in 5G networks. In this paper, we describe the rationale and background of the physical-layer technology Massive multiple-input multiple-output (MIMO), which provides the means to improve the SE of future networks by one or two orders of magnitude.

The rationale behind the Massive MIMO concept and its transmission protocol is explained from a historical

perspective and theoretic performance analysis in Section 2 and this paper concluded in Section 3.

## II. IMPORTANCE OF IMPROVING SPECTRAL EFFICIENCY

The wireless information traffic has doubled every two and a half years since the beginning of wireless communications, as observed by Martin Cooper at Array Comm in the nineties. Different technologies and use cases have dominated in different periods, but the exponential increase is currently driven by wireless data traffic in cellular and local area networks. There are no indications that this trend will break anytime soon; in fact, a slightly faster traffic growth is predicted in the well-reputed Cisco Visual Networking Index and Ericsson Mobility Report. To keep up with the rapid traffic growth, a key goal of the 5G technologies is to improve the area throughput by orders of magnitude;  $100\times$  and even  $1000\times$  higher throughput are regularly mentioned as 5G design goals. The area throughput of a wireless network is measured in bit/s/km<sup>2</sup> and can be modeled as: Area throughput (bit/s/km<sup>2</sup>) = Bandwidth (Hz)  $\times$  Cell density (cells/km<sup>2</sup>)  $\times$  Spectral efficiency (bit/s/Hz/cell). This simple formula reveals that there are three main components that can be improved to yield higher area throughput: (1) more bandwidth can be allocated for 5G services; (2) the network can be densified by adding more cells with independently operating access points; and (3) the efficiency of the data transmissions (per cell and for a given amount of bandwidth) can be improved.

The improvements in area throughput in previous network generations have greatly resulted from cell densification and allocation of more bandwidth. In urban environments, where contemporary networks are facing the highest traffic demands, cellular networks are nowadays deployed with a few hundred meters inter-site distances and wireless local area networks (WLANs) are

available almost everywhere. Further cell densification is certainly possible, but it appears that we are reaching a saturation point. Moreover, the most valuable frequency bands are below 6 GHz because these frequencies can provide good network coverage and service quality, while higher bands might only work well under short-range line-of-sight conditions. In a typical country like Sweden, the cellular and WLAN technologies have in total been allocated more than 1 GHz of bandwidth in the interval below 6 GHz and thus we cannot expect any major bandwidth improvements either.

### 2.1 Multi-User MIMO Communication

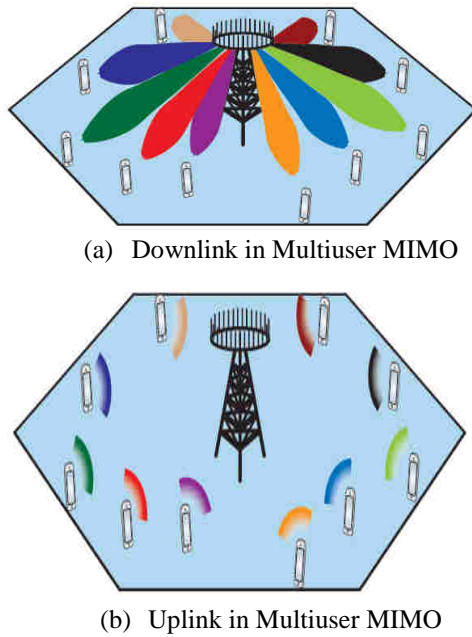
The SE of a single-input single-output (SISO) communication channel, from a single-antenna transmitter to a single-antenna receiver, is upper bounded by the Shannon capacity, which has the form  $\log_2(1+\text{SNR})$  bit/s/Hz for additive white Gaussian noise (AWGN) channels. The SISO capacity is thus a logarithmic function of the signal-to-noise ratio (SNR), denoted here as SNR. To improve the SE we need to increase the SNR, which corresponds to increasing the power of the transmitted signal. For example, suppose we have a system that operates at 2 bit/s/Hz and we would like to double its SE to 4 bit/s/Hz, then this corresponds to improving the SNR by a factor 5, from 3 to 15. The next doubling of the SE, from 4 to 8 bit/s/Hz, requires another 17 times more power. In other words, the logarithm of the SE expression forces us to increase the transmit power exponentially fast to achieve a linear increase in the SE of the SISO channel. This is clearly a very inefficient and non-scalable way to improve the SE, and the approach also breaks down when there are interfering transmissions in other cells that scale their transmit powers in the same manner. We therefore need to identify another way to improve the SE of cellular networks.

Each base station (BS) in a cellular network serves a multitude of user terminals. Traditionally, the time/frequency resources have been divided into resource blocks and only one of the user terminals was active per block. This terminal can then receive a single data stream with an SE quantified as  $\log_2(1 + \overline{\text{SNR}})$ . The efficient way to increase the SE of a cellular network is to have multiple parallel transmissions. If there are  $G$  parallel and independent transmissions, the sum SE becomes  $G \log_2(1 + \overline{\text{SNR}})$  where  $G$  acts as a multiplicative pre-log factor. Parallel transmissions can be realized by having multiple transmit antennas and multiple receive antennas. There are two distinct cases

1. Point-to-point MIMO: where a BS with multiple antennas communicates with a single user terminal having multiple antennas [39].

2. Multi-user MIMO: where a BS with multiple antennas communicates with multiple user terminals, each having one or multiple antennas [34].

There are many reasons why multi-user MIMO is the most scalable and attractive solution [17]. Firstly, the wavelength is 5-30 cm in the frequency range of cellular communication (1-6 GHz). This limits the number of antennas that can be deployed in a compact user terminal for point-to-point MIMO, while one can have almost any number of spatially separated single-antenna terminals in multi-user MIMO. This is an important distinction since the number of simultaneous data streams that can be separated by MIMO processing equals the minimum of the number of transmit and receive antennas. Secondly, the wireless propagation channel to a user terminal is likely to have only a few dominating paths, which limits the ability to convey multiple parallel data streams to a terminal in point-to-point MIMO. The corresponding restriction on multi-user MIMO is that the users need to be, say, a few meters apart to have sufficiently different channel characteristics, which is a very loose restriction that is true in most practical scenarios. Thirdly, advanced signal processing is needed at the terminals in point-to-point MIMO to detect the multiple data streams, while each terminal in multi-user MIMO only needs to detect a single data stream. The canonical multi-user MIMO system consists of a BS with  $M$  antennas that serves  $K$  single-antenna terminals in Figure 1 for a schematic illustration. The BS multiplexes one data stream per user in the downlink and receives one stream per user in the uplink. Simply speaking, the BS uses its antennas to direct each signal towards its desired receiver in the downlink, and to separate the multiple signals received in the uplink. If the terminal is equipped with multiple antennas, it is often beneficial to use these extra antennas to mitigate interference and improve the SNR rather than sending multiple data streams [6]. For the ease of exposition, this chapter concentrates on single-antenna terminals. In this case,  $\min(M, K)$  represents the maximal number of data streams that can be simultaneously transmitted in the cell, while still being separable in the spatial domain. The number  $\min(M, K)$  is referred to as the multiplexing gain of a multi-user MIMO system.



*Fig.1: Illustration of the downlink and uplink transmission in a multi-user MIMO system, where the BS is equipped with  $M$  antennas and serves  $K$  user terminals simultaneously. This illustration focuses on line-of-sight propagation where the downlink signals can be viewed as angular beams, but multi-user MIMO works equally well in non-line-of-sight conditions. (a) Downlink in multiuser MIMO. (b) Uplink in multi-user MIMO*

## 2.2 Linear Processing Schemes

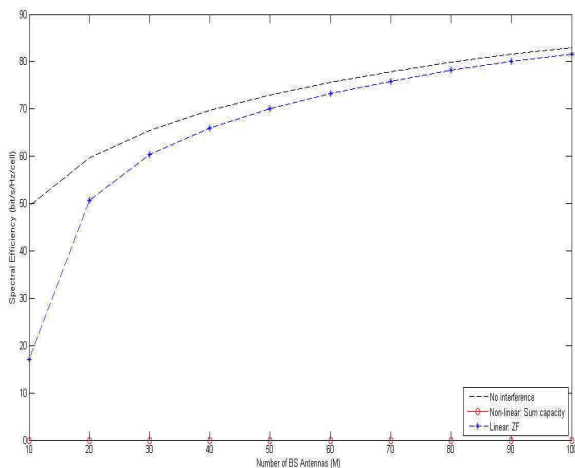
The research on multi-user MIMO, particularly with multi-antenna BSs, has been going on for decades. Some notable early works are the array processing papers [1, 38, 44, 47], the patent [36] on spatial division multiple access (SDMA), and the seminal information-theoretic works [11, 18, 42, 43, 46] that characterized the achievable multi-user capacity regions, assuming that perfect channel state information (CSI) is available in the system. In this section, we summarize some of the main design insights that have been obtained over the years.

Capacity-achieving transmission schemes for multi-user MIMO are based upon non-linear signal processing; for example, the dirty-paper coding (DPC) scheme that achieves the downlink capacity and the successive interference cancellation (SIC) scheme that achieves the uplink capacity. The intuition behind these schemes is that the inter-user interference needs to be suppressed, by interference-aware transmit processing or iterative interference-aware receive processing, to achieve the optimal performance. These non-linear schemes naturally require extensive computations and accurate CSI, because otherwise the attempts to subtract interference cause more harm than good.

How large are the gains of optimal non-linear processing (e.g., DPC and SIC) over simplified linear processing schemes where each user terminal is treated separately? To investigate this, let us provide a numerical example where  $K = 10$  user terminals are simultaneously served by a BS with  $M$  antennas. For simplicity, each user is assumed to have an average SNR of 5 dB, there is perfect CSI available everywhere, and the channels are modeled as uncorrelated Rayleigh fading. Figure 2 shows the average sum SE, as a function of  $M$ , achieved by sum capacity-achieving non-linear processing and a simplified linear processing scheme called zero-forcing (ZF), which attempts to suppress all interference.

The results are representative for both uplink and downlink transmissions. This simulation shows that the non-linear processing greatly outperforms linear ZF when  $M \approx K$ . The operating point  $M=K$  makes particular sense from a multiplexing perspective since the multiplexing gain  $\min(M, K)$  does not improve if we let  $M$  increase for a fixed  $K$ . Nevertheless, Figure 2 shows that there are other reasons to consider  $M > K$ ; the capacity increases and the performance with linear ZF processing approaches the capacity. Already at  $M = 20$  (i.e.,  $M=K = 2$ ) there is only a small gap between optimal non-linear processing and linear ZF. In fact, both schemes also approach the upper curve in Figure 2 which represents the upper bound where the interference between the users is neglected. This shows that we can basically serve all the  $K$  users as if each one of them was alone in the cell.

The performance analysis and optimization of linear processing schemes have received much attention from academic researchers. While non-linear schemes are hard to implement but relatively easy to analyze and optimize, linear processing schemes have proved to have the opposite characteristics. In particular, computing the optimal downlink linear precoding is an NP-hard problem in many cases [27], which requires monotonic optimization tools. Nevertheless, the suboptimal ZF curve in Figure 2 was generated without any complicated optimization, thus showing that the optimal linear processing obtained in [9] can only bring noticeable gains over simple ZF for  $M \approx K$ , which is the regime where we have learnt not to operate.



*Fig.2: Average spectral efficiency in a multi-user MIMO system with  $K = 10$  users and varying number of BS antennas. Each user has an average SNR of 5 dB and the channels are Rayleigh fading. The sum capacity is compared with the performance of linear ZF processing and the upper bound when neglecting all interference. The results are representative for both uplink and downlink*

As mentioned earlier, the BS needs CSI in multi-user MIMO systems to separate the signals associated with the different users. Perfect CSI can typically not be achieved in practice, since the channels are changing over time and frequency, and thus must be estimated using limited resources. The channel estimation of a frequency-selective channel can be handled by splitting the frequency resources into multiple independent frequency-flat sub channels that can be estimated separately.

A known pilot sequence is transmitted over each such sub channel and the received signal is used to estimate the channel response. In order to explore all spatial channel dimensions, this sequence must at least have the same length as the number of transmit antennas [8]. This means that a pilot sequence sent by the BS needs to have the length  $M$ , while the combined pilot sequence sent by the single-antenna user terminals needs to have the length  $K$ . There are two ways of implementing the downlink and uplink transmission over a given frequency band. In frequency division duplex (FDD) mode the bandwidth is split into two separate parts: one for the uplink and one for the downlink. Pilot sequences are needed in both the downlink and the uplink due to the frequency selective

fading, giving an average pilot length of  $\frac{(M + K)}{2}$  per

sub channel. There is an alternative time-division duplex (TDD) mode where the whole bandwidth is used for both downlink and uplink transmission, but separated in time.

If the system switches between downlink and uplink faster than the channels are changing, then it is sufficient to learn the channels in only one of the directions. This leads to an average pilot length of  $\min(M, K)$  per sub channel, if we send pilots only in the most efficient direction. In the preferable operating regime of  $M \leq K$ , we note that TDD systems should send pilots only in the uplink and the pilot length becomes  $\min(M, K) = K$ .

We conclude that TDD is the preferable mode since it not only requires shorter pilots than FDD, but is also highly scalable since the pilot length is independent of the number of BS antennas.

We give a concrete numerical example in Figure 3 for downlink transmission with  $K = 10$  users, an SNR of 5 dB, and uncorrelated Rayleigh fading channels. Two linear precoding schemes are considered; (a) maximum ratio (MR) and (b) zero forcing (ZF). This simulation compares the SE obtained when having perfect CSI with the performance when having CSI estimated with pilot sequences of length  $\tau_p$ . The SE is shown as a function of the number of BS antennas,  $M$ , and we compare TDD mode using  $\tau_p = K = 10$  with FDD mode using either  $\tau_p = 10$ ,  $\tau_p = M$ ,

or  $\tau_p = \min(M, 50)$  where the latter models an arbitrarily chosen maximum pilot length of 50 (e.g., motivated by pilot overhead constraints).

In TDD mode there is a visible performance loss in Figure 3 as compared to having perfect CSI. The loss with MR precoding is very small, which shows that it is robust to estimation errors. The performance loss is larger for ZF precoding, since estimation errors make it harder to suppress interference, but we notice that ZF anyway provide higher performance than MR for all considered  $M$ . We notice that the performance losses are substantially constant irrespective of the number of BS antennas, thus TDD systems always benefit from adding more antennas. In contrast, FDD systems only benefits from adding more antennas if the pilot sequences are also made longer, as in the case  $\tau_p = M$ . With  $\tau_p = 10$  there is no benefit from having more than 10 antennas, while the performance saturates at 50 antennas when  $\tau_p = \min(M, 50)$ .

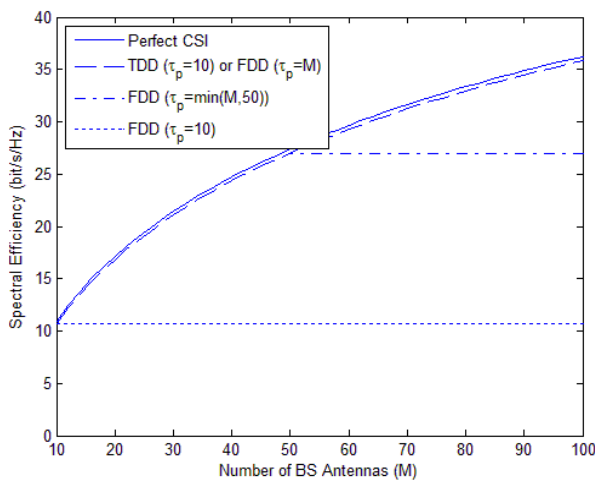
In summary, TDD operation is fully scalable with respect to the number of BS antennas, while FDD operation can only handle more antennas by also increasing the pilot overhead. It is practically feasible to deploy FDD systems with many antennas, particularly for slowly varying

channels where we can accept a large pilot overhead, but TDD is always the better choice in this respect.

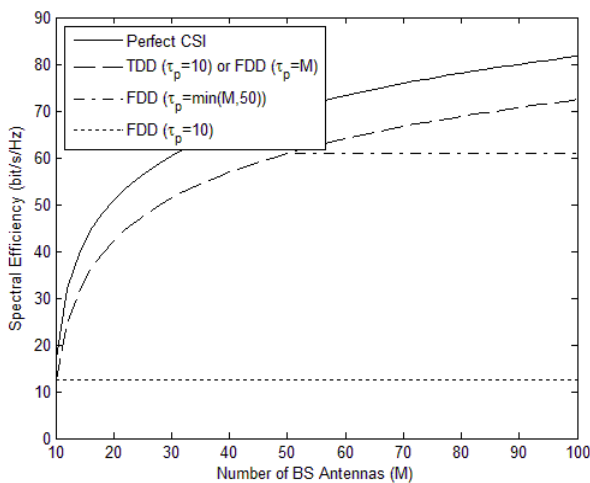
Note that the uplink works in the same way in the TDD and FDD modes, while the distinct benefit of TDD in terms of scalability appears in the downlink.

### 2.3 Favourable Propagation

Recall from Figure 2 that by adding more BS antennas, both the sum capacity achieving non-linear processing and the simplified linear ZF processing approached the case without interference. This is not a coincidence but a fundamental property that is referred to as favorable propagation.



(a) Downlink simulation with Maximum ratio Precoding



(b) Downlink with Zero forcing Precoding

Fig.3: Average downlink spectral efficiency, as a function of the number of BS antennas, with different processing schemes and different types of CSI available at the BS. (a) Downlink simulation with maximum ratio precoding. (b) Downlink simulation with zero-forcing precoding

Let  $h_1, h_2 \in \mathbb{C}^M$  represent the channel responses between a BS and two different user terminals. If these vectors are non-zero and orthogonal in the sense that

$$h_1^H h_2 = 0 \tag{1}$$

Where  $(\cdot)^H$  denotes the conjugate transpose, then the BS can completely separate the signals  $s_1, s_2$  transmitted by the users when it observes  $y = h_1 s_1 + h_2 s_2$ . By simply computing the inner product between  $y$  and  $h_1$ , the BS obtains

$$h_1^H y = h_1^H h_1 s_1 + h_1^H h_2 s_2 = \|h_1\|^2 s_1 \tag{2}$$

Where the inter-user interference disappeared due to (1) the same thing can be done for the second

user  $h_2^H y = \|h_2\|^2 s_2$ . Note that the BS needs perfect

knowledge of  $h_1$  and  $h_2$  to compute these inner products. The channel orthogonality in (1) is called favorable propagation, since the two users can communicate with the BS without affecting each other.

Is there a chance that practical channels offer favorable propagation? Probably not according to the strict definition that  $h_1^H h_2 = 0$ , but an approximate form of favorable propagation is achieved in non-line-of-sight scenarios with rich scattering

**Lemma 1:** Suppose that  $h_1 \in \mathbb{C}^M$  and  $h_2 \in \mathbb{C}^M$  have

independent random entries with zero mean, identical distribution, and bounded fourth-order moments, then

$$\frac{h_1^H h_2}{M} \rightarrow 0 \tag{3}$$

almost surely as  $M \rightarrow \infty$ .

This lemma shows that the inner product between  $h_1$  and  $h_2$ , if normalized with the number of BS antennas, goes asymptotically to zero as  $M$  increases. We refer to this as asymptotic favorable propagation and note that this phenomenon explains the behaviors in Figure2; the difference between having no inter-user interference and suppressing the interference by ZF becomes smaller and smaller as the number of antennas increases, because the loss in desired signal gain when using ZF reduces when the user channels become more orthogonal.

One special case in which Lemma 1 holds is  $h_1, h_2 \in \mathbb{C}^M \sim \mathcal{N}(0, I_M)$ , where  $\mathcal{N}(\cdot, \cdot)$  denotes a multi-variate circularly symmetric complex Gaussian distribution and  $I_M$  is the  $M \times M$  identity matrix. This is known as uncorrelated Rayleigh fading and in this case

one can even prove that the variance of the inner product in (3) is  $1/M$  and thus decreases linearly with the number of antennas [31]. Many academic works on Massive MIMO systems consider Rayleigh fading channels, due to the analytic tractability of Gaussian distributions. Nevertheless, Lemma 1 shows that asymptotic favorable propagation holds for other random channel distributions as well. This mathematical result can be extended to also include correlation between the elements in a channel vector. One can also derive similar analytic results for line-of-sight propagation [31] and behaviors that resemble asymptotic favourable propagation have been observed also in the real-world multi-user MIMO channel measurements presented in [16, 20].

This is yet another reason to design multi-user MIMO systems with  $M \gg K$ . It is, however, important to note

that  $\frac{(h_1^H h_2)}{M} \rightarrow 0$ , as  $M \rightarrow \infty$  does not imply

that  $h_1^H h_2 \rightarrow 0$ . Strict favorable propagation is unlikely to appear in practical or theoretical channels. In fact, the inner product  $h_1^H h_2$  grows roughly as  $\sqrt{M}$  for Rayleigh fading channels. The key point is that this correlation has a negligible impact, since the SE depends on

$\frac{(h_1^H h_2)}{M} \rightarrow$  which goes to zero roughly as  $\frac{1}{\sqrt{M}}$ .

Moreover, the main suppression of inter-user interference appears already at relatively small number of antennas due to the square root.

### 2.3 Massive MIMO Concept

The Massive MIMO concept was proposed in the seminal paper [28] and described in the patent [29], both of which have received numerous scientific awards. Massive MIMO takes multi-user MIMO communications to a new level by designing a highly scalable communication protocol that utilizes the described in Sect. 2.2. The basic information and communication theoretic limits of this 5G technology were established in early works such as [3, 19, 21, 23, and 30]. In this paper we define Massive MIMO as follows:

Massive MIMO is a multi-user MIMO system with  $M$  antennas and  $K$  users per BS. The system is characterized by  $M \gg K$  and operates in TDD mode using linear uplink and downlink processing.

This definition does not manifest any particular ratio between  $M$  and  $K$ , or any particular orders of magnitude that these parameters should have. One attractive example is a system with  $M$  in the range of 100 to 200 antennas, serving between  $K = 1$  and  $K = 40$  users depending on the

data traffic variations. The first public real time implementation of Massive MIMO is the LuMaMi testbed described in [41], which features  $M = 100$  and  $K = 10$ . We stress that other definitions of Massive MIMO are available in other works and can both be more restrictive (e.g., require certain dimensionality of  $M$  and  $K$ ) and looser (e.g., also include FDD mode), but in this chapter we only consider the definition above.

The BS antenna array typically consists of  $M$  dipole antennas, each having an effective size  $\lambda/2 \times \lambda/2$ , where

$\lambda$  is the wavelength. This means that an array area of  $1 \text{ m}^2$  can fit 100 antennas at a 1.5 GHz carrier frequency and 400 antennas at 3 GHz. Each antenna is attached to a separate transceiver chain, so that the system can access the individual received signal at each antenna and select the individual signals to be transmitted from each antenna. The array can have any geometry; linear, rectangular, cylindrical, and distributed arrays are described in [25]. It is important to note that no model of the array geometry is exploited in the Massive MIMO processing, thus the antennas can be deployed arbitrarily without any geometrical array calibration.

The basic Massive MIMO transmission protocol is illustrated in Figure 4. The time frequency resources are divided into blocks of size  $B_c$  Hz and  $T_c$  seconds, with the purpose of making each user channel approximately frequency-flat and static within a block. Hence, the bandwidth  $B_c$  is selected to be smaller or equal to the anticipated channel coherence bandwidth among the users, while  $T_c$  is smaller or equal to the anticipated channel coherence time of the users. For this particular reason, each block is referred to as a coherence interval. The number of transmission symbols that fit into a coherence interval is given by  $\tau_c = B_c T_c$ , due to the Nyquist-Shannon sampling theorem. The dimensionality of the coherence interval depends greatly on the anticipated system application. For example, a coherence interval of  $\tau_c = 200$  symbols can be obtained with  $B_c = 200 \text{ kHz}$  and  $T_c = 1 \text{ ms}$ , which supports highway user velocities in urban environments at 2 GHz carrier frequencies. Much larger coherence intervals (e.g.,  $\tau_c$  at the order of  $10^3$  or  $10^4$ ) can be obtained by limiting the application to scenarios with low user mobility and short delay spread.

Each coherence interval is operated in TDD mode and can contain both downlink and uplink payload transmissions. To enable channel estimation at the BS,  $\tau_p$  of the symbols in each coherence interval are allocated for uplink transmission of pilot sequences (where

$\tau_p \geq K$ ), while the remaining  $\tau_c - \tau_p$  symbols can be allocated arbitrarily between uplink and downlink payload data transmissions

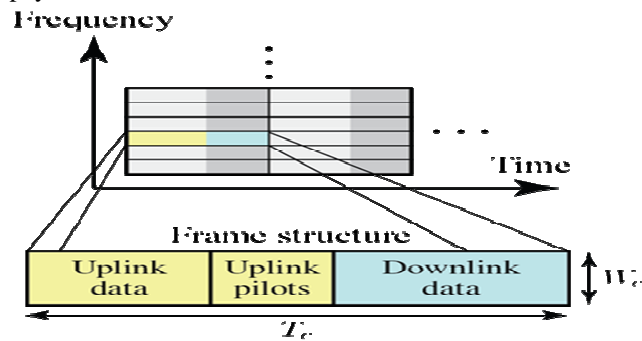


Fig.4: Illustration of the basic Massive MIMO transmission protocol, where the time-frequency resources are divided into coherence intervals, each containing  $\tau_c = B_c T_c$  transmission symbols. Each coherence interval contains uplink pilot sequences and can be used for both uplink and downlink payload data transmission based on TDD operation

### III. CONCLUSION

In this paper we conclude that the linear processing techniques such as ZF provides a sum spectral efficiency close to the sum capacity when  $M \gg K$ . The channel estimation is simplified when operating in TDD mode, since the pilot sequences only need to be of length  $K$  irrespective of the number of BS antennas  $M$ . Also most of the wireless channels seem to provide asymptotic favorable propagation conditions only.

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